FORECASTING, SIMULATION AND PIP CONTROL OF INTERURBAN TRAFFIC NETWORKS

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Abstract

This paper discusses a number of research areas concerned with dynamic traffic management, including forecasting, simulation and, ultimately, control of motorway traffic flow. A detailed non-linear simulation of the Amsterdam A10 west ring road, utilising a dynamic state space representation called the Statistical Traffic Model (STM) is described. The simulation is employed in the development of a robust, co-ordinated ramp metering system, based on the Non-Minimal State Space (NMSS) approach to control design, which yields adaptive Proportional-Integral-Plus (PIP) controllers. Finally, when employed in predictive mode, the STM requires forecasts of the on-ramp inputs to the motorway. One approach to obtaining such univariate forecasts, namely Dynamic Harmonic Regression (DHR) is also discussed.

1 Introduction

The electronics revolution has lead to enormous advances in transportation technology. In particular, communication advancements coupled with induction loops implanted in road surfaces, allow data on traffic processes from distant points in a large network to be utilised in real time. However, with the huge volume of real time measurements available from instrumented networks, automatic techniques of data processing, together with intelligent systems for effectively utilising the information, are now required. This paper discusses a number of research areas concerned with the problem, including forecasting, simulation and control.

A framework for dynamic state-space modelling, which transforms on-line roadside measurements of traffic flow into explicit assessments of the current and future state of an inter-urban network, has recently been developed [1-2]. Here, basic ideas of traffic flow, such as conservation of vehicles, are built into a Statistical Traffic Model (STM). Although developed primarily to provide short term forecasts with a rolling horizon of fifteen-minute intervals, the STM also provides a good simulation model of the traffic system suitable for control system design purposes. In this regard, the present paper describes the development of STM based Matlab/Simulink™ computer simulations (section 3).

Previous papers (e.g. [3]) have introduced Proportional-Integral-Plus (PIP) control, which has been successfully implemented in a number of difficult applications, including the multivariable control of climate in a large horticultural glasshouse [4]. In line with previous experience of such complex non-linear systems, the present paper shows how the essential behaviour of the traffic system can be approximated well by simple, time variable parameter (TVP), linear transfer function models, which have been obtained statistically by reference to both the STM and real data from the road measurement system. Such linear models define the Non-Minimal State-Space (NMSS) equations that describe the system completely and are required for PIP control design (section 4).

Linear Quadratic (LQ) optimisation is employed to determine the feedback control law, in a manner similar to the approach described in [5]. However, the resulting PIP controllers make efficient use of all the useful measurements available from the motorway system, avoiding the need for complex techniques such as state reconstruction and loop transfer recovery, so enhancing considerably the robustness of the resultant control system to uncertainties in the models and the system. These new modelling and control techniques are employed in the control of traffic dynamics on the Amsterdam ring road, using the STM simulation as a surrogate for the real system. In order to bring the results as close to real traffic conditions as possible, the study employs measured data, available from monitored roads in The Netherlands, as inputs to the STM system.

In fact, when employed in predictive mode, the STM requires short term forecasts of these on-ramp flows, which are then used as inputs to the non-linear equations. Therefore, in section 2 below, the paper discusses a fully recursive, TVP methodology which appears particularly appropriate for traffic data, characterised as it is by a mixture of smooth gradual changes over a day, as well as by more rapid fluctuations that can take place during congestion. In particular, a dynamic harmonic regression (DHR) model [6-7] is utilised to forecast the highly periodic traffic flow data, with its twice daily rush hour cycles. Here, the seasonal frequencies are estimated from the autoregressive spectrum (or sample periodogram), and these then define the characteristics of the DHR model.

2 Forecasting traffic variables

The STM [1-2] includes the following components: a state vector that holds two variables per link, namely traffic density (veh/km) and flow (veh/min), ordered by the corresponding measurement station; an observation vector that holds information from the monitoring stations; a vector of exogenous inputs consisting of the flows from the on-ramp entrances; and, finally, variables that specify the behaviour of the system, such as measures of link length and capacity. The state transition matrix depends on traffic dynamics, which classically follow differential equations analogous to those of fluid dynamics, as discussed in section 3 below. The network is built up from a large number of links, i.e. the sections of motorway between two adjacent measurement stations. The simplest component is thus a single segment of motorway, typically 0.5 to 1 km long. Special equations describe joins and forks, so that on-ramps, off-ramps and intersections can all be modelled.

The Kalman Filter is employed as a data assimilation device for estimating the current state of the process and for forecasting. However, the STM itself requires short term forecasts of the inputs to the non-linear equations, including on-ramp and boundary (upstream) flows. For example, [2] discusses the performance of the STM when applied to traffic data collected from the A12 between Zoetermeer and The Hague. This is a small network that can be described by 29 state variables, but which includes 11 on-ramps. In this context, the present paper discusses the application of a univariate methodology for forecasting the traffic variables, based on a DHR model. Although motivated by the requirements of the STM, the forecasting approach described here has certain advantages its own right [6-7].

2.1 Dynamic Harmonic Regression (DHR)

The forecasting method discussed here exploits an extremely flexible approach to nonstationary time series analysis, based on the definition of a DHR model [6-7]. The model is particularly useful for adaptive seasonal adjustment, signal extraction, interpolation over gaps, and forecasting. In the case of the traffic variables, the main seasonality is the twice daily rush hour cycle (for Monday to Friday), although an additional weekly periodicity may also be identified. The seasonal frequencies are estimated from the autoregressive spectrum or sample periodogram.

The DHR model, which is a special case of a univariate unobserved components model, is represented as follows,

$$y_t = T_t + S_t + e_t \tag{1}$$

where y_t is the observed time series; T_t is a trend or low frequency component; S_t is a seasonal component; and e_t is a normally distributed Gaussian sequence with zero mean and variance σ^2. In order to allow for nonstationarity in the time series, the various components in (1) can be characterised by TVPs, with each TVP defined as a nonstationary stochastic variable, as discussed below. Here, the most important unobserved component is the seasonal term S_t:

$$S_t = \sum_{i=1}^{R} \left\{ a_{it} \cos\left(\omega_i t\right) + b_{it} \sin\left(\omega_i t\right) \right\} \tag{2}$$

where a_{it} and b_{it} are TVPs; and ω_i are a set of R frequencies chosen by reference to the spectral properties of the time series. In general, each of these a_{it} and b_{it} parameters, as well as the trend component T_t are defined by a two dimensional stochastic state vector $\boldsymbol{x}_{it} = [l_{it} \quad d_{it}]^T$, where l_{it} and d_{it} are, respectively, the changing level and slope of the associated trend or TVP. The stochastic evolution of $\boldsymbol{x}_{it}$ is assumed to be described by a Generalised Random Walk (GRW) process of the form [6].

$$\boldsymbol{x}_{it} = \boldsymbol{F}_i \boldsymbol{x}_{it-1} + \boldsymbol{G}_i \eta_{it} \ ; \ \boldsymbol{F}_i = \begin{bmatrix} \alpha & \beta \\ 0 & \gamma \end{bmatrix} ; \ \boldsymbol{G}_i = \begin{bmatrix} \delta \\ 1 \end{bmatrix} \tag{3}$$

where η_t is an 2 dimensional vector containing zero mean, white noise inputs. These white noise inputs are assumed to be independent of the observation noise e_t and have a diagonal covariance matrix $\boldsymbol{Q}$. This general model includes two well known special cases, namely the Random Walk (RW: $\alpha = \delta = 0$; $\beta = \gamma = 1$); and the Integrated Random Walk (IRW: $\alpha = \beta = \gamma = 1; \delta = 0$). The overall state space model is constructed by the aggregation of the subsystem matrices defined in (2), with the observation equation defined by equation (1) as described in [6-7].

2.2 Time Variable Parameter (TVP) estimation

The estimation of the TVPs is based on two step (prediction-correction) Kalman filtering, accompanied by optimal fixed interval smoothing procedures [7]. In these algorithms, the diagonal *Noise Variance Ratio* (NVR) matrix $\boldsymbol{Q_r}$ is defined as follows,

$$\boldsymbol{Q}_r = \boldsymbol{Q} / \sigma^2 \tag{4}$$

Within the DHR context, these NVR parameters are unknown and need to be estimated before the filtering and smoothing algorithms can be utilised. Their estimation by a special form of optimisation in the frequency domain is discussed in [7].

Note that forecasting, interpolation and backcasting are an inherent part of the filtering and smoothing algorithms: if missing data anywhere within, or immediately outside, the data set are detected, then the filtering and smoothing algorithms simply replace the missing samples by their expectations, based on the DHR model and the data. In the case of the filtering algorithm, this yields the multi-step ahead forecasts; while the smoothing equations provide the interpolation over gaps within the data set.

2.3 DHR forecasting of traffic flow

To illustrate the methodology described above, 15 minute average flow measurements (i.e. vehicles per minute) for the A12 motorway in the Netherlands are studied. Such data, collected during a week in May 1994 for a particular location on the main carriageway, are illustrated in figure 1. The twice daily rush hour cycles are clear evidence of seasonality in the data, supporting the use of DHR techniques. The AR-spectrum of the data in figure 1 confirms the 24 hour (96 samples per cycle) periodicity, by correctly identifying the harmonics of the series. The first 13 harmonics (with periods of 96, 96/2, ..., 96/13 samples) are selected for the DHR model, since these are particularly well defined and have relatively high amplitudes in the AR-spectrum.

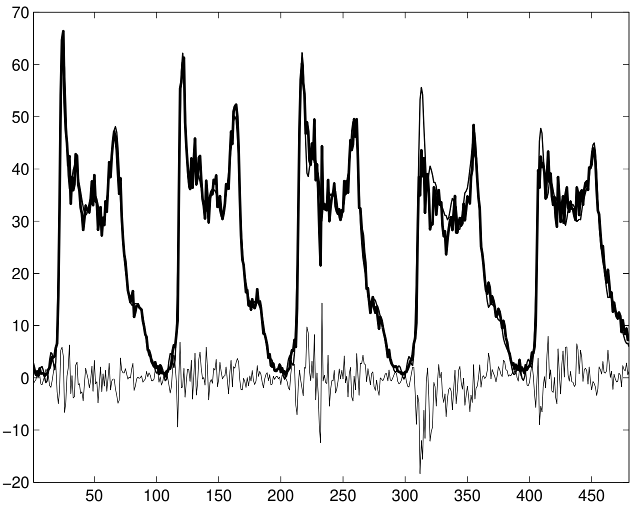

Figure 1. Traffic flow data (veh/min, 15 min averages, thick trace); smoothed DHR model and (from sample 192) 1-step ahead forecasts (thin trace); modelling/forecasting errors (bottom trace); all plotted against sample number.

So far in the study, only one week of data has been available and so the NVRs have been optimised by fitting the AR-spectrum of the model to only a sub-set of the weekly data from Monday to Tuesday, leaving the remainder for testing the forecasting ability of the model. Figure 1, shows the smoothed model fit for these two days, together with the 1-step ahead forecast for the rest of the week. The DHR model yields a 11% reduction in the mean absolute forecasting errors over this period compared to a simple persistence forecast. As expected, the improvement is much greater for longer forecasting intervals: for example, with a 4 step (or 1 hour ahead) prediction, the DHR model yields a 48% reduction in the forecasting errors compared to persistence.

Clearly, the DHR model employed in this preliminary investigation does not include all of the relevant dynamics since the morning rush hour peaks on Thursday and Friday are considerably lower than those earlier in the week, resulting in an over estimate of the flow at these times (although it is worth noting that the TVPs have adapted to the changing dynamics by Friday, yielding a better forecast than for a constant parameter model).

To improve the model, more harmonics must be included when more historic data become available: the lower rush hour peaks on the last two days of figure 1, for example, may be part of a longer term trend that could be accommodated automatically if several weeks of data were analysed. A more comprehensive study, taking account of such observations, is presently in progress. This new research is also looking at forecasting 1 minute sampled data.

3 STM motorway simulation

The present section discusses how the STM is converted from a forecasting tool [1-2], to a simulation of inter-urban traffic networks, using the Amsterdam ring road A10 west motorway, immediately south of the Coen tunnel, as an example. The three types of equations which comprise the STM are as follows: a balance equation, which ensures that vehicles are conserved; a flow equation, based on the identity $q = v\, r$; and the ubiquitous, highly nonlinear, fundamental traffic diagram (FTD). In the simplest case, a *minimum velocity* flow equation may be employed, as follows (although alternative approaches are presently being explored),

$$q_{s,t+1} = \alpha_s \min\left(v_{s,t}, v_{s+1,t}\right) r_{s,t} \tag{5}$$

where $q_{s,t}$ is the flow at time t and location s; $r_{s,t}$ is the density; $v_{s,t}$ is the velocity obtained from the non-linear FTD; and α_s is a parameter for location s. The balance equation takes the form,

$$r_{s,t+1} = r_{s,t} + k_s^{-1}\left(q_{s-1,t} - q_{s,t}\right) \tag{6}$$

where k_s is the length of the link (multiplied by the number of lanes) divided by the sample time. Similar equations hold for joins and forks, with the latter requiring the estimation of a turning fraction between the two downstream choices.

Clearly, consistent units should be employed throughout the simulation, with α_s and k_s modified to ensure any necessary conversions. Similarly, care has to be taken over the sampling rates employed. For the A10 west, a time sampling rate of 10 seconds is found to be most appropriate for the simulation, given that the length of the sections varies from 0.4 to 0.8 km. Too coarse or too fine a sampling interval results in numerical instability problems. Finally, the structure of the FTD used in the present simulations is illustrated in figure 2.

The simulation version of the STM is solved using the Matlab/Simulink™ package. Here, the equations of each STM module (a link, join or fork) are assembled as a grouped element, or sub-system, in Simulink. These sub-systems are then connected by lines that carry the variables used in the equations. The A10 west simulation, for example, which includes the 4 on-ramps immediately south of the Coen tunnel, consists of 6 link blocks, 4 joins and 3 forks, as illustrated in figure 3. The Coen tunnel is represented by the link on the far right of the figure. Note that the parameters α_s and k_s, as well as the exact shape of the FTD, differ for the various sections, depending, for example, on the number of lanes.

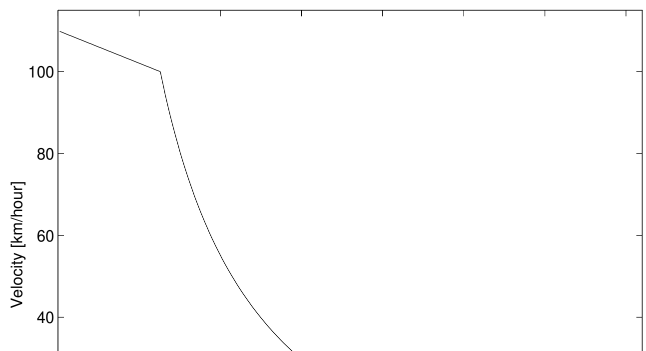

Figure 2. Typical fundamental traffic diagram.

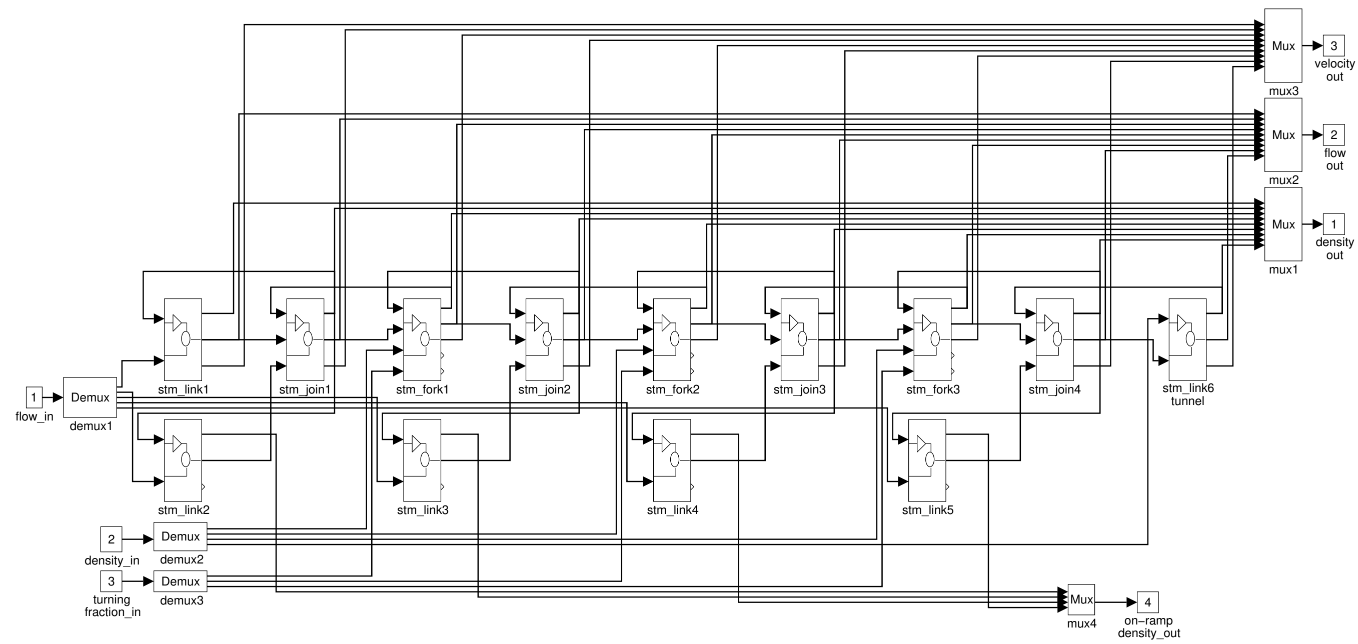

Figure 3. STM simulation of north bound traffic on the A10 west, south of the Coen tunnel.

Figure 4 illustrates the response of the simulation to a particular set of realistic conditions. Here, the velocity of each of the sections is plotted on the same graph, and it is clear that for most of the time, the traffic is in the free flow linear part of the FTD, with the velocities in the range 100 to 110 km/hour (see figure 2). However, the upstream input employed in the simulation is a measured time series from the appropriate location on the A10 and includes the morning rush hour. During the peak flows, therefore, the density of traffic on the corridor has built up so that, eventually, the critical level is exceeded and there is a temporary breakdown in flow, resulting in the lower velocities in the middle part of the figure. Note that, in this particular case, only those locations close to the Coen tunnel are effected. This is a result of traffic slowing down as they enter the tunnel (which is represented by a modified FTD for that link).

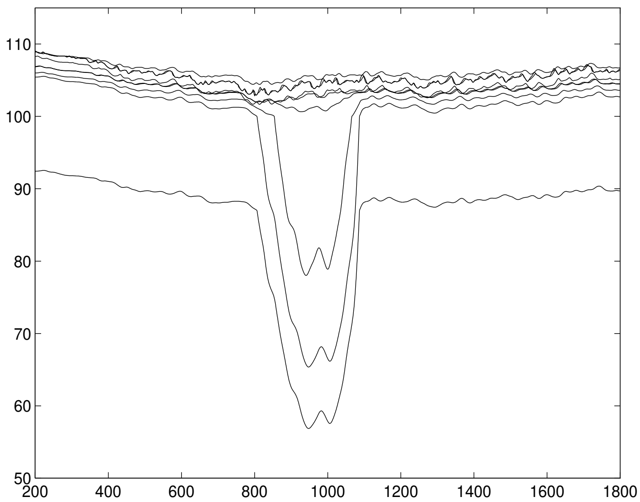

Figure 4. Simulated velocity (km/hour) plotted against 10 second samples.

4 Control of the A10 west simulation

This section reviews the essential ideas behind the control of traffic in a given localised region of the network and discusses its practical embodiment in the co-ordinated local PIP controller. Although most researchers working in this area mention broad aims, such as the desire to reduce congestion, delays, travel times, pollution, and fuel consumption, the specific control objectives typically involve maintaining a specified flow or density. For example, in [5] ramp metering (traffic lights on slip roads) is employed to ensure that the density of traffic at three locations above a bottleneck on the Boulevard Peripherique in Paris, do not exceed certain critical levels (which might lead to congestion, as in figure 4). These are the same type of set points employed in the present paper.

The possibility of employing optimally chosen set points, perhaps as part of a multilayered, hierarchical controller is discussed in, for example, [8-10]. A similar approach to this is described in [11], where it is suggested that a 'diversion policy', which satisfies the 'origin-destination' requirements of users, could be complemented by local level controllers (including ramp-metering).

4.1 Local Linear Model (LLM)

In order to develop a PIP control algorithm, a linearised representation of the traffic system is required. The *small perturbation* behaviour of the non-linear traffic system can be approximated well by a simple linearised TVP transfer function model, which is identified and estimated from either the non-linear STM or from measured traffic data. This *Local Linear Model* takes the following simple form for the flow,

$$q_{s,t} = a\, q_{s,t-1} + b\, q_{s-1,t-1} + c\, q_{s+1,t-1} \tag{7}$$

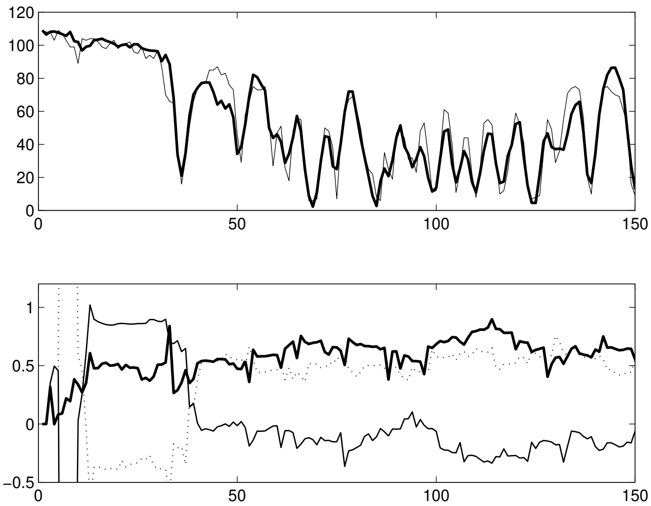

Figure 5. Local linear model (based on the final parameter estimates, thick) fitted to measured velocity data (km/hour) from the A2 (May 1990) with 1 minute sampling (top); recursive updates of a (dotted), b (thin) and c (thick).

where a, b and c are the TVP's. Models with an identical structure to this may also be estimated for velocity and density. The identification and estimation of the LLM exploits the special properties of the Simplified Refined Instrumental Variable (SRIV) algorithm [12]. Because of the inherent time variability of the parameters in (7), a *self adaptive* controller is required in practice. Such on-line adaptive control, where the parameters of the LLM are estimated recursively in order to take account of changes in traffic conditions, is fairly straightforward to implement since the prior off-line analysis also utilises recursive estimation in the form of the SRIV algorithm. For example, figure 5 illustrates the recursive updates of the three parameters for a particular location on the A2 motorway in The Netherlands, in this case for velocity data.

Note that from the 35*th* sample onwards, the data in figure 5 shows evidence of congestion. This is reflected in the rapid adjustment to relatively large values for the downstream c parameter (compared to the upstream b parameter) as the troughs in the flow travel *upstream* during congestion. Figure 5 thus illustrates how the TVPs adapt to changing conditions and demonstrates how the model can be fitted to measured data from real networks. In fact, when the final parameter values are applied to the whole data set, the coefficient of determination, $R_T^2 = 0.92$ implies a very good fit.

Finally, note that, although (7) does not represent traffic dynamics sufficiently well for forecasting or simulation studies, this is *not* its purpose: it is employed only in the design of the PIP control algorithm. However, the *evaluation* of the PIP system is based on applying this algorithm to the much more sophisticated non-linear STM which, in this context, is regarded as a surrogate for the real traffic system.

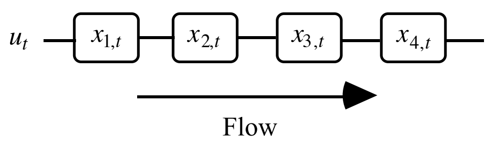

Figure 6. Control of a 4 link corridor.

4.2 Proportional-Integral-Plus (PIP) control

The PIP algorithm is based on the definition of suitable NMSS equations [3-4], which are derived directly from the estimated parameters of the LLM. Here the NMSS state vector $\boldsymbol{x}_t$, is composed of *present and past values of the measured input and output signals*, together with an *integral of error* state z_t to ensure inherent type-one servomechanism performance associated with the selected output (i.e., this output will always achieve its desired set point $y_d(k)$ under equilibrium conditions in the closed loop).

Consider, as an example, the simplified motorway corridor illustrated in figure 6. Here, the objective is to employ an on-ramp at the upstream end of the corridor, in order to control the flow at the third downstream link. By controlling the level at the third link in this manner, the intervening flows automatically behave well because of the natural linkages in the system. In this NMSS description, the control input u_t is defined as the total of the upstream and on-ramp flow,

$$\begin{aligned} \boldsymbol{x}_t &= \boldsymbol{F}\boldsymbol{x}_{t-1} + \boldsymbol{g}u_{t-1} + \boldsymbol{d}y_t^d \\ y_t &= \boldsymbol{h}\boldsymbol{x}_t \end{aligned} \tag{8}$$

$$\boldsymbol{x}_t = \begin{bmatrix} x_{4,t} & x_{3,t} & x_{2,t} & x_{1,t} & z_t \end{bmatrix}^T$$

$$\boldsymbol{F} = \begin{bmatrix} a_{4,t} & b_{4,t} & 0 & 0 & 0 \\ c_{3,t} & a_{3,t} & b_3,t & 0 & 0 \\ 0 & c_{2,t} & a_{2,t} & b_2,t & 0 \\ 0 & 0 & c_{1,t} & a_{1,t} & 0 \\ -c_{3,t} & -a_{3,t} & -b_{3,t} & 0 & 1 \end{bmatrix}; \boldsymbol{g} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_1,t \\ 0 \end{bmatrix}; \boldsymbol{d} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Having established a suitable NMSS model for the system, the appropriate gains defining the PIP state variable feedback (SVF) control law can be determined by optimisation of a standard linear quadratic (LQ) cost function (or the equivalent Linear-Quadratic-Gaussian, LQG, formulation in the stochastic case). This approach can be considered as a more sophisticated version of the Linear-Quadratic-Integral (LQI) algorithm in [5].

With this formulation, it is found that the PIP controller offers considerable improvements to a standard tuned PI (Proportional-Integral) controller applied to the same system (i.e. STM simulation of figure 6). For example, when a disturbance is simulated in the middle of the corridor, the local PI controller has no way of reacting to the problem (because it does not 'see' the effects of the disturbance unless the back-up of traffic is sufficient to be detected just downstream of the on-ramp). By contrast, the PIP implementation smoothly backs off such a disturbance, by reducing the on-ramp flow accordingly [8].

4.3 PIP control of the A10 west simulation

Computer-based simulation models are a well established means of testing new approaches to traffic control before their implementation on real networks. In this tradition, PIP designs are presently being evaluated on the detailed Matlab/Simulink™ simulation of the A10 west corridor illustrated in figure 3. The objective is to ensure that the density of north bound traffic, south of the Coen tunnel, does not exceed critical levels that lead to congestion. The final algorithm includes recursive LLM estimators for each link, and determines the flow inputs for each of the 4 on-ramps. Following the approach of [11], a number of special scenarios are being considered, including peak hour periods, off-peak periods and traffic incidents, in order to ensure that the control algorithm can cope with such realistic events. However, earlier investigations which were part of an initial feasibility study reported by [8-10], suggest that PIP controllers are indeed robust to such events.

5 Conclusions

This paper has briefly reviewed a dynamic state-space modelling technique, called the Statistical Traffic Model (STM), which provides transport management with a 'data assimilation' tool for monitoring, prediction and control. However, the research has established that a much simpler, Local Linear Model (LLM) can represent traffic data to the accuracy required for adaptive control system design. As a consequence, it is possible to linearise the essential dynamics of the non-linear traffic system and so represent a given region of the traffic network in a linearised, Non-Minimal State-Space (NMSS) form. This means that standard state variable feedback techniques can be applied to the traffic control problem, to yield self adaptive Proportional-Integral-Plus (PIP) ramp metering controllers which are able to maintain excellent local control of the traffic variables.

It is clear that, in designing this local level of control, a higher layer in the hierarchical control system is required in order to specify suitable set points, based on operating the STM in a predictive mode. In this situation, however, the STM requires short term forecasts of the uncontrolled on-ramp flows, which are then employed as inputs to the non-linear equations. Therefore, the paper has also discussed the application of a fully recursive, time variable parameter estimation methodology based on a dynamic harmonic regression (DHR) model. In addition to forecasting, the technique automatically interpolates over missing samples, a very useful ability when dealing with traffic data, which can be of highly erratic quality.

The importance of the co-ordinated local PIP controllers, within this wider context, is that the higher level objectives will only be effective if there is tight control at the local level. An integrated traffic management system, using the Amsterdam ring road as an example, is presently under development at Lancaster University within the framework of the European Community DACCORD project.

The advantage of the PIP approach is that it has great flexibility in design terms. For example, it can be applied directly to single and multivariable systems described by backward shift [4], continuous-time, or delta operator models. In addition, the PIP control system is ideal for incorporation within a multi-objective optimisation framework, where satisfactory compromise can be obtained between conflicting objectives and where difficult control objectives, such as multivariable decoupling control, can be achieved by optimisation of the weighting matrices in the LQ optimal PIP cost function.

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