ROBUST PIP CONTROL OF MULTIVARIABLE STOCHASTIC SYSTEMS

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Abstract

This paper discusses the development of robust versions of multivariable Non-Minimal State Space (NMSS) design procedures, for Proportional-Integral-Plus (PIP) control systems. The formulation allows for the exploitation of any state variable feedback control procedure, including modern robust designs based on H_2 and H_∞ methods, as well as risk sensitive optimisation. The control designs are evaluated on a multivariable coupled drive rig and the *IFAC93* benchmark problem.

1 Introduction

An important practical consideration in control system design, is the robustness of the control system performance to stochastic influences arising from both parametric uncertainty and stochastic disturbance inputs to the system. This paper discusses the development of robust versions of multivariable Non-Minimal State Space (NMSS) design procedures, for the Proportional-Integral-Plus (PIP) control systems previously introduced in [1-2]. Robust control design aims to ensure good closed loop performance under difficult conditions, such as model uncertainty and 'worst case' disturbance inputs.

In this regard, the practical utility of the PIP controllers are evaluated on two systems, namely a multivariable coupled drive rig [3] and the *IFAC93* benchmark [4]. The first of these examples is a laboratory scale plant representing a materials handling system, where control of speed and tension is required; while the latter is a stochastic simulation, whose parameters vary randomly within specified ranges.

The methodological approach follows from earlier research, in which NMSS forms are formulated so that, in the deterministic situation, full state feedback PIP control can be implemented directly from the measured input and output signals of the controlled process, without resort to the design and implementation of a state reconstructor (or observer). This yields a full state variable feedback (SVF) design which is naturally robust to uncertainty and eliminates the need for measures such as Loop Transfer Recovery. Moreover, the inherent SVF formulation allows for the exploitation of any SVF control procedure, including state space solutions of modern robust designs based on H_2 and H_∞ methods. For example, unlike a conventional minimal state space controller, the NMSS formulation ensures that the *risk sensitive* Linear-Exponential-of-Quadratic-Gaussian (LEQG) control methodology [5] is easily implemented in practice, utilising the 'complete observations' form of the algorithm.

The format of the paper is as follows. Section 2 reviews multivariable NMSS/PIP design and discusses optimisation of the LQ weighting matrices; while section 3 considers robustness and filtering polynomials. Section 4 describes PIP risk sensitive control applied to the coupled drives rig. Methods of assessing the robustness of the various designs are discussed in section 5; and PIP control of the *IFAC93* benchmark problem is discussed in section 6. Finally, the conclusions are presented in section 7.

2 Multivariable NMSS/PIP design

Previous papers have introduced multivariable NMSS/PIP design methods, which have been successfully implemented in a range of applications (e.g. [2-3]). Consider the following p-input, p-output, discrete time system represented in terms of the left matrix fraction description,

$$\begin{aligned}
&\boldsymbol{y}(k)=[\boldsymbol{A}(z^{-1})]^{-1}\boldsymbol{B}(z^{-1})\boldsymbol{u}(k) \qquad (1)\\
&\boldsymbol{A}(z^{-1})=\boldsymbol{I}+\boldsymbol{A}_1 z^{-1}+\ldots+\boldsymbol{A}_n z^{-n} \quad (\boldsymbol{A}_n\neq\boldsymbol{0})\\
&\boldsymbol{B}(z^{-1})=\boldsymbol{B}_1 z^{-1}+\ldots+\boldsymbol{B}_m z^{-m} \quad (\boldsymbol{B}_m\neq\boldsymbol{0})
\end{aligned}$$

Here, $\boldsymbol{A}_i\,(i=1,2,\ldots,n)$ and $\boldsymbol{B}_i\,(i=1,2,\ldots,m)$ are p by p matrices. If necessary, some of the initial $\boldsymbol{B}$ terms could take null value to accommodate pure time delays in the system. The non-minimal state space (NMSS) representation associated with the above model is defined by the following state vector

$$\boldsymbol{x}(k)^T=\left[\boldsymbol{y}(k)^T\cdots\boldsymbol{y}(k-n+1)^T\ \boldsymbol{u}(k)^T\cdots\boldsymbol{u}(k-m+1)^T\ \boldsymbol{z}(k)^T\right] \qquad (2)$$

where $\boldsymbol{y}(k)$ and $\boldsymbol{u}(k)$ are the vectors of system outputs and control inputs respectively; while the *integral-of-error* state vector $\boldsymbol{z}(k)=[\boldsymbol{y}_d(k)-\boldsymbol{y}(k)]/(1-z^{-1})$, where $\boldsymbol{y}_d(k)$ is the command input vector, each element being associated with the appropriate system output. With these definitions, the NMSS representation is then defined directly in terms of the following discrete-time state equation,

$$\boldsymbol{x}(k+1)=\boldsymbol{F}\boldsymbol{x}(k)+\boldsymbol{G}\boldsymbol{u}(k)+\boldsymbol{D}\boldsymbol{y}_d(k+1) \qquad (3)$$

where $\boldsymbol{F}$, $\boldsymbol{G}$ and $\boldsymbol{D}$ are defined in [2]. Type-1 servomechanism control is automatically accommodated by the introduction of the integral-of-error vector $\boldsymbol{z}(k)$. Consequently, if the closed-loop system is stable, then steady-state decoupling is inherent in the design. The state variable feedback (SVF) control law is then defined in the normal manner, i.e.,

$$\boldsymbol{u}(k) = -\boldsymbol{L}\boldsymbol{x}(k) \tag{4}$$

where,

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{L}_0 \; \boldsymbol{L}_1 \cdots \boldsymbol{L}_{n-1} & \boldsymbol{M}_1 \cdots \boldsymbol{M}_{m-1} & -\boldsymbol{K}_I \end{bmatrix} \tag{5}$$

is the SVF control gain matrix. The NMSS control strategy results in a control system which can be related structurally to more conventional designs, such as multivariable PI and PID controllers, as illustrated in figure 1. However, because it exploits fully the power of SVF, it is inherently much more flexible and sophisticated, allowing for SVF control strategies such as closed loop pole assignment or optimisation of a standard LQ cost function,

$$J = \sum_{k=1}^{\infty} \left\{ \boldsymbol{x}(k)^T \boldsymbol{Q}\boldsymbol{x}(k) + \boldsymbol{u}(k)^T \boldsymbol{R}\boldsymbol{u}(k) \right\} \tag{6}$$

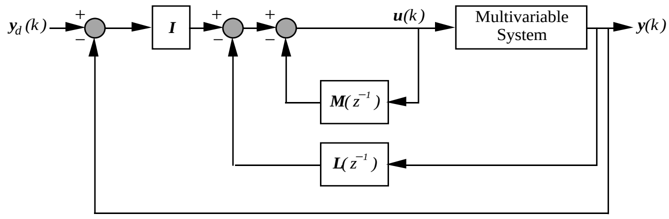
The weighting matrices in this cost function are not readily translated into typical closed loop response characteristics, such as decoupling, overshoot and rise times. In fact, manual tuning of $\boldsymbol{Q}$ and $\boldsymbol{R}$ is usually limited to the diagonal weights. However, the PIP control system is ideal for incorporation within a multi-objective optimisation framework, where satisfactory compromise can be obtained between conflicting objectives such as robustness and multivariable decoupling, by concurrent optimisation of the diagonal and off diagonal elements in the weighting matrices of the LQ cost function [6].

This approach compares with that of Fleming-Pachkevitch (FP) [7], who directly optimise the control *gains* themselves. By contrast, the method described in [6] maintains the versatility and flexibility of the FP technique, but since it is the LQ *weights* that are optimised, has the advantage of generating only guaranteed stable optimal solutions, thus allowing for better (smoother) defined optimisation problems.

Finally, as discussed in [8-9], it is straightforward to develop multivariable *generalised PIP* (GPIP) controllers, with various structural modifications which can enhance the practical implementation. These include: command input anticipation (which can counteract non-minimum phase effects); feedforward of deterministic disturbances; feedforward compensation for stochastic disturbances; and measurement noise attenuation.

3 Filtering and robustness polynomials

The previous discussion has concentrated on the deterministic model of the system (1). However, since it is formulated in state space terms, the NMSS approach to control system design can easily be converted to an optimal stochastic form by extending the LQ solution to the Linear-Quadratic-Gaussian (LQG) form; i.e. by invoking the separation theorem and introducing an optimal Kalman filter (KF) for state estimation [10].



Output feedback polynomial matrix	$\boldsymbol{L}(z^{-1}) = \boldsymbol{L}_0 + \boldsymbol{L}_1 z^{-1} + \cdots + \boldsymbol{L}_{n-1} z^{-n+1}$
Input feedback polynomial matrix	$\boldsymbol{M}(z^{-1}) = \boldsymbol{M}_1 z^{-1} + \cdots + \boldsymbol{M}_{m-1} z^{-m+1}$
Integral Control	$\boldsymbol{I} = \boldsymbol{k}_I (1 - z^{-1})^{-1}$

Figure 1. Multivariable PIP block diagram.

Note, however, that the NMSS approach to control design does not *need* any form of state reconstruction since it always involves full state feedback which is based solely on the measured input and output signals.

In essence, therefore, while the KF is only required for noise attenuation in PIP systems, it carries with it the disadvantage of decreased robustness: in particular, the state estimation is too dependent upon the input-driven model of the system and the cancellation of the observer polynomial [11]. For this reason, it is often advantageous to utilise different, nominally sub-optimal, approaches to noise attenuation which do not involve the model-based state reconstruction of the KF or equivalent observer. For example, the non-minimal states can be specially prefiltered or additional filters can be introduced into the forward path of the PIP control structure prior to NMSS design.

In the latter case, the PIP control system exploits a slightly modified NMSS representation, as described in [12]. Here, the state vector is formed from past *filtered* outputs and past and present *filtered* inputs, although the *unfiltered* integral of error state is still included in order to ensure type one servo-mechanism performance. In the univariate case, this approach involves the introduction of a filter polynomial:

$$T(z^{-1}) = 1 + t_1 z^{-1} \cdots + t_n z^{-n} \tag{7}$$

Judicious choice of this filter polynomial allows for control designs with varying degrees of robustness to both measurement noise and parametric uncertainty. For example, if the system is stable, then it is often desirable to choose $T(z^{-1}) = A(z^{-1})$. The filtered output and input are then defined as,

$$y^*(k) = {y(k)}\big/{T(z^{-1})} \quad ; \quad u^*(k) = {u(k)}\big/{T(z^{-1})} \tag{8}$$

These and other alternatives, which may be compared with existing techniques such as *Loop Transfer Recovery,* are currently being evaluated. In general, they seem to be advantageous, yielding improved noise rejection with acceptable reductions in gain and phase margins.

4 Risk sensitive control

Over the last decade, the topic of robust control and, in particular, H_∞ control theory has received considerable attention. H_∞ control ensures that bounds on the H_∞-norm of certain transfer functions, from disturbance signals to particular variables of interest, are achieved. Unlike the LQG formulation, however, a particular H_∞ criterion does not yield a unique solution. One approach, for example, is the minimum entropy solution of Mustafa and Glover [13]; which, in turn, is closely related to the Linear-Exponential-of-Quadratic-Gaussian (LEQG) cost function considered in the present paper [5]. This relationship means that the LEQG formulation has certain robustness properties that enhance its potential importance to control engineers. Here, the cost function involves the expectation of the exponential of the standard LQ criterion (6),

$$\gamma(\theta) = -2\theta^{-1} \log_e E\left\{e^{-\theta J/2}\right\} \tag{9}$$

where θ is the (real scalar) *risk sensitive* parameter. It is clear that, in the deterministic case, $\gamma(\theta)$ reduces to the conventional (*risk neutral*) LQ cost function. Otherwise LEQG optimal control may be formulated in one of two ways: either in a *risk-averse* (pessimistic) manner (which relates to H_∞ control), where disturbances are assumed to work against the controller ($\theta < 0$); or, alternatively, as a *risk-preferring* (optimistic) controller, where it is assumed that uncertainty advantages the controller ($\theta > 0$).

In the PIP-LEQG case, a backwards Riccati recursion [5] is employed to determine the control algorithm, based on a multivariable stochastic NMSS system developed in [14]. It should be stressed that, unlike the conventional minimal state space case, the special NMSS equations employed here ensure that it is possible to implement the 'complete observations' solution to the LEQG problem, making the technique very straightforward to implement in practice.

For example, consider the regulation of a multivariable coupled drives rig [3]. The PIP-LEQG algorithm yields good decoupling control, as illustrated in figure 2 for the case when $\theta = 10$. A positive θ generally yields LEQG responses with smoother input signals than the LQ case ($\theta = 0$), at the expense of a corresponding increase in the rise time. By contrast, when a negative θ is employed, LEQG control yields a faster closed loop response with a noisier input signal. However, as discussed in [8, 14] it is possible to approximately replicate the response of a risk sensitive solution by simply optimising the weights of the standard LQ cost function. The robustness consequences of these results and the relationship with H_∞ control in the NMSS/PIP case is presently being investigated.

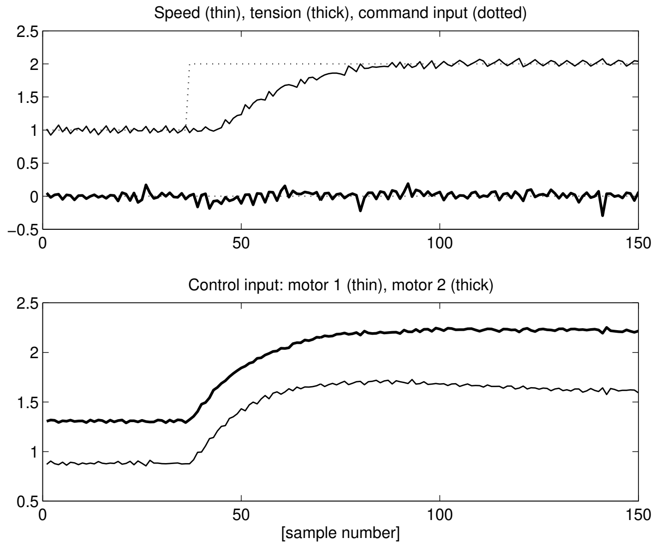


Figure 2. PIP-LEQG implementation results from the coupled drives rig, showing the response to a step change in speed.

5 Measures of robustness

The various methods discussed above may be evaluated in both time and frequency domains by taking advantage of Monte Carlo (MC) analysis, which utilises estimates of the parametric uncertainty obtained from the SRIV identification algorithm [15]. Furthermore, the new multi-objective optimisation methods mentioned in section 2 above, can be employed in conjunction with MC analysis to improve the performance and robustness of the final PIP designs.

With the current wide availability of powerful desktop computers, MC analysis provides one of the simplest and most attractive approaches to assessing the sensitivity of a controller to parametric uncertainty. Here, the model parameters for each realisation in the MC analysis, are selected randomly from the joint probability distribution defined by the parametric covariance matrix $\boldsymbol{P}$ [15], and the sensitivity of the PIP controlled system to parametric uncertainty is evaluated from the ensemble of resulting closed loop response characteristics. The various ensemble results obtained in this manner are normally provided in graphical form: for instance, plots of the closed loop step, impulse and frequency responses; and plots of the closed loop pole positions ('stochastic root loci').

In addition to the standard frequency domain tools (such as Bode and Nyquist plots), which are mainly limited to univariate systems, methods developed for H_∞ robust control design can be used to provide measures of sensitivity and robustness for multivariable PIP control design. The sensitivity function $S(e^{-j\omega t})$ and complementary sensitivity function $T(e^{-j\omega t})$ for the closed loop system, provide information on the tracking performance and sensitivity to disturbances and measurement noise. The infinity norms ($\|.\|_\infty$) of $S(e^{-j\omega t})$ and $T(e^{-j\omega t})$ are used to calculate robust gain margins and phase margins, and in the univariate case, they can be shown to be related to the radii of the M-circles, i.e. the closest approaches of the open loop Nyquist plot to the critical ($-1, j0$) point. MC simulation analysis may be used in conjunction with these frequency methods, as illustrated by [16]. Finally, robust H_∞ optimisation in the NMSS/PIP context is presently under investigation.

6 Control of the IFAC93 benchmark

Although only a single input, single output (SISO) plant, the *IFAC93* benchmark [4] is a difficult problem since it includes stringent closed-loop performance requirements, despite the fact that the parameters of the plant are known only within a certain range. In fact, the plant operates at three different stress levels, with higher stress levels implying greater time variations of the unknown parameters.

The set point is specified as a square wave varying between +1 and -1 with a period of 20 seconds, while an under/overshoot of up to 0.2 is allowed. Occasional excursions of the output within the range -1.5 to 1.5 are acceptable, as long as these extreme limits are *never* exceeded. Furthermore, the rise time should be designed as fast as possible given the stress levels, but 2 or 3 seconds with rapid settling should be aimed for. Finally, the plant input saturates at -5 and +5. The complete, continuous time (s-operator) transfer function of the plant, together with the range of parameter values for each stress level is given in [4].

The final control designs are evaluated on a stochastic "black box" simulation available in the form of uncompiled C code. Here, the user algorithm replaces the pre-programmed unity feedback controller. Using this simulation, it is recommended that the plant is simulated over at least 300 seconds (or by repeated simulation over shorter times) to obtain a representative picture of the variations. However, it should be pointed out that, due to noise and parameter variations, the response will be different every time the program is run.

6.1 Standard PIP-LQ control of the benchmark

To obtain the control model (1), the plant is simulated in open loop and data collected at a sampling rate of 0.5 seconds. SRIV estimation coupled with the YIC identification criterion [15], yields the following first order discrete time transfer function representation of the system,

$$y(k) = \frac{-0.0526z^{-1} + 0.1469z^{-2}}{1 - 0.9059z^{-1}} u(k) \tag{10}$$

The weights in the PIP-LQ formulation are initially defined by assuming that the state weighting matrix is a $(n+m)$ by $(n+m)$ diagonal matrix,

$$\boldsymbol{Q} = diag\left[q_1 \; q_2 \; \dots \; q_{n+m}\right] \tag{11}$$

where the user defined output weighting parameters $q_1, q_2, \dots, q_n$ and input weighting parameters $q_{n+1}, q_{n+2}, \dots, q_{n+m-1}$ are set equal to common values of q_y and q_u, respectively; while q_{n+m} is denoted by q_e to indicate that it provides a weighting constraint on the integral of error state variable $z(k)$. In the SISO case, the 'default' PIP controller, which usually yields a good performance in practice, is that obtained using total optimal control weights of unity, i.e., $q_y = 1/n$, $q_u = 1/(m-1)$, and $q_e = 1$. Similarly, the scalar (in the SISO case) input weight $\boldsymbol{R}$ is set equal to $1/(m-1)$.

However, for the *IFAC93* benchmark these weights yield a relatively slow response, whereas by setting $q_e = 10$ the control objectives stated above are clearly satisfied, as illustrated in figure 3. The response shown in figure 3 is obtained by simulating the continuous time transfer function defined by [4], with the parameter uncertainty set to zero. To assess the robustness of the design, the continuous time model is simulated at stress level 2 with the positive and negative extremes of each of the 5 parameters listed by [4], i.e. 32 possible realisations. Although it is possible that more extreme responses may be obtained with certain combinations of the parameters in their intermediate range, the 32 simulations employed here provide a good guide to the performance of the controller, before it is finally implemented on the black box simulation.

When the PIP controller employed in figure 3, is applied to the continuous time system with parametric uncertainty the results are poor, as illustrated in figure 4. Here, the maximum overshoot (i.e. the worst case from all 32 simulations) is 1.75, well over the specified limit; while the control input becomes saturated in many of the realisations. To improve the performance of the controller, multi-objective optimisation is employed as discussed in section 6.2 below.

It is worth pointing out that, in all these simulations, when the input reaches the +/- 5 limit, it is this saturated level that is actually employed; however, in such cases, the incremental form of the PIP controller means that integral wind-up does not occur, although clearly the performance of the controller may suffer in other ways. Finally, in the present section, note that the PIP control algorithm is implemented in a *forward path* form, since this is found to yield the best results for this system. For a full discussion of the incremental, forward path PIP control structure see, for example, references [8-9].

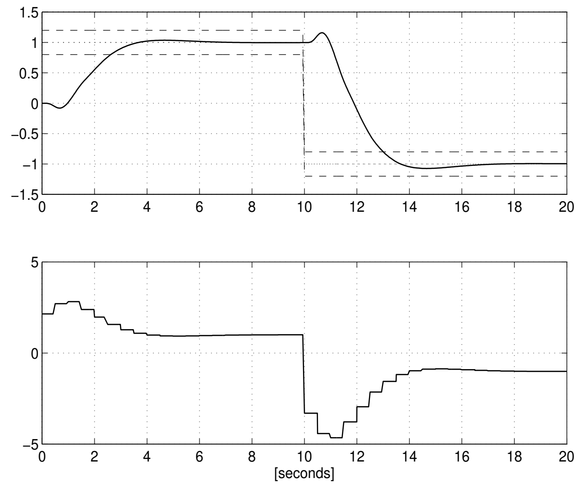


Figure 3. PIP-LQ control of the continuous time plant; output (solid trace) and (dashed) +/- 0.2 of the set point (top); control input (bottom).

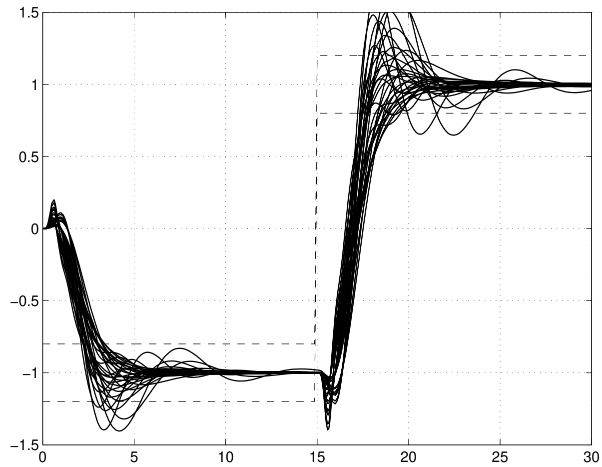


Figure 4. PIP-LQ control (with the same gains as for figure 3) of the continuous time plant, with extremes of parametric uncertainty at stress level 2; 32 simulations.

6.2 Optimisation for different stress levels

The present section employs multi-objective optimisation to improve the robustness of the PIP controller. All the results are obtained using the Matlab™ optimisation toolbox. Because of space limitations, the discussion is limited to stress level 2, however, the LQ weights may be similarly optimised for stress levels 1 and 3.

To start with, consider the optimisation of just two goals, namely the maximum overshoot and the time it takes for the output to reach 0.8 of the set point. The primary objective in this exercise is to minimise the overshoot, however, the rise time is a necessary constraint since, without it, the routines will very quickly find a solution that has an extremely slow response. To minimise computation time, the optimisation is simplified by considering only the worst case scenario for each of these two goals, i.e. the two particular realisations that yield the highest overshoot and longest rise time when the default PIP-LQ controller is employed, as illustrated in figure 5. The thin trace in figure 5 is obtained with default LQ weights, i.e. $\boldsymbol{Q} = diag\begin{bmatrix}1 & 0.5 & 1\end{bmatrix}$, while the improved thick trace is with the optimised weights thus:

$$\boldsymbol{Q} = \begin{bmatrix} 4.51 & 1.09 & 1.53 \\ 1.09 & 0.42 & 0.72 \\ 1.53 & 0.72 & 2.06 \end{bmatrix} \tag{12}$$

yielding a control gain vector,

$$\boldsymbol{K} = \begin{bmatrix} 5.71 & 0.93 & -1.17 \end{bmatrix} \tag{13}$$

The control input weight $\boldsymbol{R}$ is set to the default value of 0.5 in both cases. When employed over the full 32 realisations, the optimised PIP algorithm yields a considerable improvement over the initial design as illustrated in figure 6.

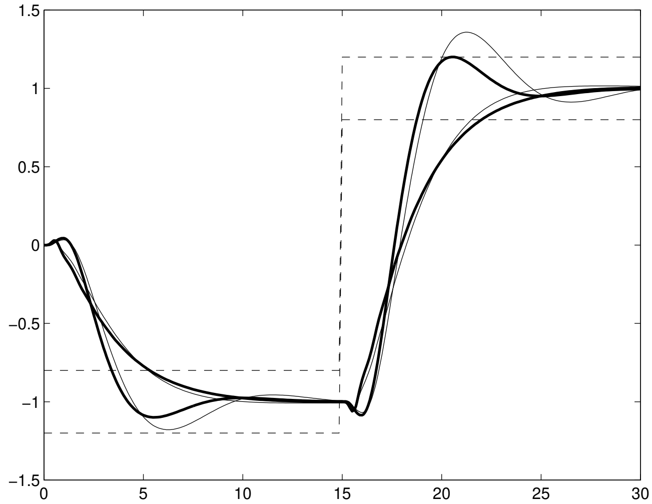


Figure 5. PIP-LQ control of the continuous time plant at stress level 2; dashed lines show +/- 0.2 of the set point; control using default LQ weights (thin) and optimised LQ weights (thick); 2 simulations, i.e. those with extremes of overshoot and rise time.

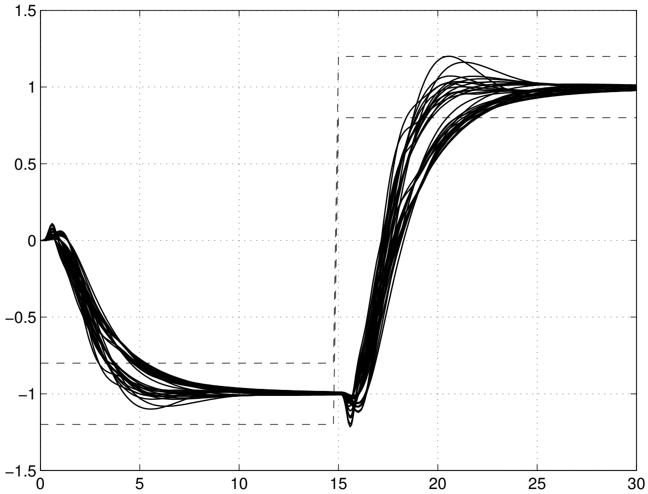


Figure 6. Optimised PIP-LQ control of the continuous time plant with extremes of parametric uncertainty at stress level 2; dashed lines show +/- 0.2 of the set point; 32 simulations; as for figure 4 except with optimised LQ weights.

For the present example, one additional goal is the minimisation of the non-minimum phase behaviour exhibited by the system. Again, only the worst case realisation (found with the default controller) is employed in the optimisation, which, therefore, now includes 3 simulations at each iteration. The new optimised control gain vector with the fastest rise time (for the worst case realisation) is as follows:

$$\boldsymbol{K} = \begin{bmatrix} 8.15 & 1.32 & -2.08 \end{bmatrix} \tag{14}$$

and this yields a response that satisfies the original control objectives stated above, for all 32 simulations. Clearly, further improvements in the design may be obtained by including even more goals and more simulations in the optimisation process, at the expense of increased computational effort. However, the simplified approach employed here, works very well for the present example.

6.3 Evaluation on black box simulation

A forward path, incremental form [6] of the PIP algorithm utilizing the control gains (14), is programmed in C and included in the code for the "black box" *IFAC93* benchmark. Figure 7 illustrates 15 realisations of the simulation, each of 20 seconds length and overlaid in the figure. These example results compare favourably with those in figure 23 of reference [4].

It is worth pointing out that continuous time versions of PIP control [17], based on SRIV estimated s-operator models, have been optimised and implemented on the benchmark, with equally successful results. Finally, note that in the present example, the optimisation has made use of the available continuous time plant model with known parameter limits, effectively a validated simulation of the system. In situations where such a model is not known, similar techniques may be employed utilising the estimated parameter uncertainty available in the P matrix, i.e. optimisation of the MC realisations.

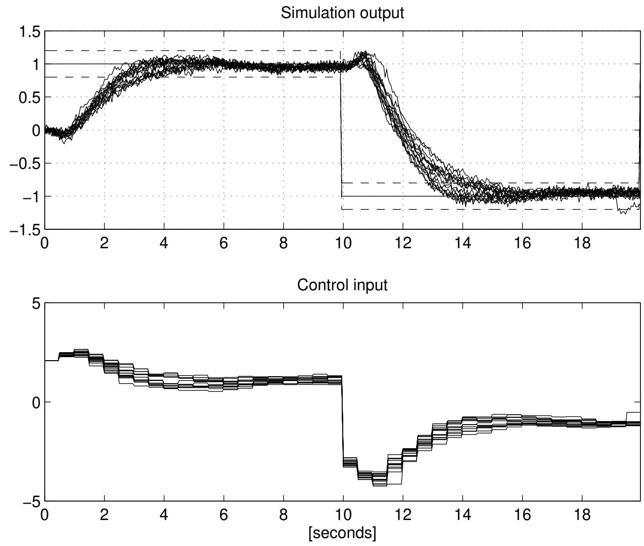


Figure 7. Optimised PIP-LQ control of the *IFAC93* benchmark at stress level 2; 15 realisations.

7 Conclusions

Previous publications have shown that the Proportional-Integral-Plus (PIP) controller has great flexibility in design terms. For example, it can be applied directly to single and multivariable systems described by backward shift [2], continuous-time [17] or delta [10] operator models. The present paper has demonstrated how, because of its non-minimal state space (NMSS) formulation, it can be extended easily to handle stochastic systems and is ideal for incorporation within a multi-objective optimisation framework. Moreover, the approach allows for the exploitation of any state variable feedback control procedure, including the state space solutions of modern robust designs based on risk sensitive and H_∞ methods.

In practical terms, the paper has shown how PIP risk sensitive control, employing a Linear-Exponential-of-Quadratic-Gaussian (LEQG) cost function, may be applied successfully to a multivariable coupled drives rig. In addition, the flexibility introduced by the multi-objective optimisation form of the PIP design method has been illustrated by the design of PIP controllers for the *IFAC93* benchmark problem. Here, fast robust control of the benchmark is achieved at any stress level, despite severe parametric uncertainty.

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